Séminaire MaMuX
Mathématiques, musique et relations avec d'autres disciplines

Théorie des mots en musique / Word Theory in Music

Mardi 4 mai 2010

de 14h à 18h30

Ircam, Salle Shannon
1, place I. Stravinsky 75004 Paris
(Entrée libre dans la mesure des places disponibles)

Cette séance « hors-programme » du Séminaire MaMuX est consacrée aux rapports entre théorie des mots et musique. Après une première utilisation des grammaires formelles dans l'informatique musicale dans les années 1980 [2], cette démarche a connu récemment un renouvellement d'intérêts dans la communauté des théoriciens de la musique à la fois comme un outil de formalisation de résultats connus dans la tradition diatonique américaine [3] mais également comme une approche nouvelle en théorie mathématique de la musique [4, 6, 7, 8, 9]. L'objectif de cette séance informelle est principalement celui de faire le point sur les perspectives ouvertes par l'utilisation des méthodes issues de la théorie des mots dans la formalisation algébrique des structures et processus musicaux. Elle est organisée dans le cadre du projet PEPS Interactions Maths/ST2I « Géométrie de l'Interaction et Musique » (http://recherche.ircam.fr/equipes/repmus/mamux/PEPS-GdIM.html).

Programme :

- 14h00 - 14h30 **Thomas Noll** – Fundament progressions, tonality and the word "bab" (joint work with Karst de Jong).
- Discussion
- 14h45 – 15h15 **David Clampitt** – Regions, Central Words, and *Théorème de Lucas*
- Discussion
- 15h30 – 16h00 **Norman Carey** – Locally Symmetric Series: Palindromes and Musical Scales
- Discussion

- 16h30 - 17h00 **Jean-Paul Allouche** – Inconstancy of discrete curves and sequences (joint work of J.-P. Allouche and L. Maillard-Teyssier)
- Discussion
- 17h15-17h45 **Thomas Noll** - A Specific Extension of Christoffel-Duality to a Certain Class of Sturm Numbers and their Characteristic Words (joint work with Manuel Dominguez).
- Final discussion

Résumés :

Thomas Noll (ESMuC / University of Berlin) – Fundament progressions, tonality and the word "bab" (joint work with Karst de Jong)

A side remark in Norman Carey and David Clampitt's (1989) article on well-formed scales draws a link between what they name the *structural scale* and Hugo Riemann's *tonic-subdominant-dominant* relationship. Basically, this link interprets the roots of the tonic, subdominant and dominant triads in terms of a well-formed fifth-generated 3-tone-scale. Since I studied this article I puzzle about the music-theoretical, psychological, cultural interpretation of the hierarchy of fifth-generated well-formed scales and of the well-formedness property. One particular idea took possession of mine: If the interpretation of the diatonic scale as an instance of a well-formed scale is of music-theoretical or even psychological importance, the same should be the case for its predecessors in the hierarchy. Therefore it appeared reasonable to follow this side remark. The standard mode bab of the structural scale as well as its anti-standard (and Christoffel) companion abb serve as skeletons for the traditional music-theoretical construction of the diatonic scale (b = perfect fourth, a = major second). This means: the traditional construction is compatible with the construction via Sturmian morphisms. In a word-theoretic refinement of well-formed scale theory David Clampitt and I extended the well-formedness property (via Christoffel-Duality) to all conjugates. The dual words have music-theoretical interpretations in terms of fifth/fourth foldings. The work by Karst de Jong and myself is mainly concerned with the application of these foldings in the analysis of fundament progressions in tonal harmony.

David Clampitt (The Ohio State University, USA) - Regions, Central Words, and *Théorème de Lucas*.

In our 1996 *Journal of Music Theory* article, "Regions: A Theory of Tonal Spaces in Early Medieval Treatises," Norman Carey and I defined certain maximal sets (largest *connected* sets of *complements*) called *regions*. Our definition was motivated by the musical facts that, in the appropriate contexts, regions 1) contained highly structured scales, such as *well-formed* pentatonic or diatonic scales, and 2) coincided with constructions of medieval theorists (Guido of Arezzo, Pseudo-Odo, the writers of the *Enchiriadis* treatises) as they attempted to understand the diatonic pitch world of chant. We have subsequently learned that regions exemplify *central words*, certain doubly periodic words which are maximal in length under conditions of the Fine-Wilf Theorem, and that moreover well-formed scales are conjugates of Christoffel words. Carey and I used a result on linear congruences from the 1891 *Théorie des nombres* by Edouard Lucas, which I will present in a word-theoretical context. The discussion sets the stage for Carey's Lambda word.

Norman Carey (CUNY Graduate Cente, USA) - Locally Symmetric Series: Palindromes and Musical Scales

Carey and Clampitt's "Regions" (*JMT*, 1996) utilizes the diagram in Plato's *Timeaus* as a locus for musical scales in Pythagorean tuning. The diagram suggests a two-dimensional representation of the set of (non-negative) powers of 2 and 3 and their multiples. This set is also subject to a one-dimensional representation, beginning, 1, 2, 3, 4, 6, 8, 9.... The sequence of ratios between adjacent pairs of this ordering (2/1, 3/2, 4/3, 3/2, 4/3, 9/8...) can be converted to an infinite word ("Lambda") on an infinite alphabet (abcbcd...). The number of occurrences of each letter is finite and can be shown to be derived from the continued

fraction of $Log_2(3)$. The distribution pattern for each set of letters is similar for all. The word contains all central words with slope ~$Log_2(3)$. These are all palindromes, and Lambda has a stunning array of palindromes. Palindromes in Lambda contain no more than three different letters. Most notably, between any two adjacent appearances of the same letter lies a palindrome. Finally, Lambda is an instance of an infinite class of such words.

Jean-Paul Allouche (CNRS-LRI, Université Paris-Sud) - Inconstancy of discrete curves and sequences (joint work of J.-P. Allouche and L. Maillard-Teyssier)

We recall an old result due to Cauchy and to Crofton on the average number of intersections of a random line and a given curve, and we show how to deduce a measure of "complexity" (or of "fluctuation") of a discrete curve or of a sequence of real numbers, that we call "Inconstancy". We study the inconstancy of classical sequences. We discuss possible applications to biology, stockmarket, music...

Thomas Noll (ESMuC, University of Berlin) – A Specific Extension of Christoffel-Duality to a Certain Class of Sturm Numbers and their Characteristic Words (joint work with Manuel Dominguez).

If one combines the traditional concept of pitch height with the pythagorean interval lattice (generated by fifth and octave), a dual concept to pitch height is directly motivated from the involvement of the Sturmian sequence of slope $\log_2(3/2)$: David and I call it *pitch width*. The pythagorean sturmian sequence one has an interpretation as in infinite trajectory in terms of ascending fifths and descending fourths in Z^2 approximating the width axis. To every semiconvergent of $\log_2(3/2)$ there is a dual basis of the pythagorean interval lattice: the primary step and the comma of the associated well-formed scale. And analogously there is an infinite trajectory in terms of primary and secondary scale steps approximating the height axis. And there is an associated Sturmian sequence of some slope. The research which I would like to present was motivated by the desire to understand the relation between the two slopes. The work eventually departed from the original example towards a class of Sturm numbers where the above construction yields a true duality. These numbers have purely periodic continued fraction expansions and the periods are palindromes of even length. The duality can be expressed as a reversal of the associated halfperiods (of odd length).

Références :

[1] E. Lucas, *Théorie des nombres* (tome premier), Jacques Gabay, Paris, 1891 (reprint 1991).
[2] M. Chemillier, "Monoïde libre et musique", *RAIRO Inf. Theo.*, vol. 21, n° 3 et 4, p. 341-371 et 379-417, 1987.
[3] N. Carey & D. Clampitt, "Aspects of Well-Formed Scales", *Music Theory Spectrum*, 11(2), 1989, p. 187-206.
[4] N. Carey & D. Clampitt, "Regions: A Theory of Tonal Spaces in Early Medieval Treatises", *Journal of Music Theory*, 40(1), 1996, p. 113-147.
[5] M. Lothaire, *Algebraic Combinatorics on Words*, Cambridge University Press, 2002.
[6] Th. Noll, "Sturmian sequences and morphisms: a music-theoretical application", Société Mathématique de France, Journée annuelle, 2008, pp. 79-102.
[7] D. Clampitt, M. Dominguez & Th. Noll, "Plain and Twisted Adjoints of Well-Formed Words", *Proceedings of the Second International Conference MCM 2009*, Elaine Chew, Adrian Childs and Ching-Hua Chuan (Eds.), Springer, 2009, p. 65-80.
[8] D. Clampitt & Th. Noll, "Regions and Standard Modes", *Proceedings of the Second International Conference MCM 2009*, Elaine Chew, Adrian Childs and Ching-Hua Chuan (Eds.), Springer, 2009, p. 81-92.
[9] D. Clampitt & Th. Noll, "Modes, the Height-Width Duality, and Handschin's Tone Character", *Music Theory Online,* 16(4), 2010. Draft Version online at : http://user.cs.tu-berlin.de/%7Enoll/HeightWidthDuality.pdf

Planning du séminaire :

- Samedi 10 octobre 2009 : Géométrie de l'information et musique.
- Vendredi 13 novembre 2009 : Géométrisation de la logique et de l'informatique musicale.
- Vendredi 4 décembre 2009 : Approche fonctorielle en informatique musicale
- Samedi 5 décembre 2009 : école mathématique pour musiciens et autres non-mathématiciens animée par Pierre Cartier
- Vendredi 15 janvier 2010 : Théorie des nœuds et musique
- Vendredi 12 mars 2010 : Représentations pour l'informatique musicale. Graphes et S-langages
- Samedi 13 mars 2010 : école mathématique pour musiciens et autres non-mathématiciens animée par Pierre Cartier
- Vendredi 9 avril 2010 : Espaces de Chu et musique
- *Mardi 4 mai 2010 (séance exceptionnelle) : Théorie des mots en musique - Word Theory in Music*
- Vendredi 14 mai 2010 : Musique algorithmique
- Samedi 15 mai 2010 : école mathématique pour musiciens et autres non-mathématiciens animée par Pierre Cartier

Contacts :

Le Séminaire est organisé par L'Equipe Représentations Musicales de l'IRCAM, en collaboration avec Guerino Mazzola (MultiMediaLab de Université de Zürich / School of Music, University of Minnesota), Franck Jedrzejewski (CEA Saclay - INSTN/UESMS), Thomas Noll (Escola Superior de Musica de Catalunya) et avec le soutiens du CNRS (UMR STMS - Sciences et technologies de la musique et du son).

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