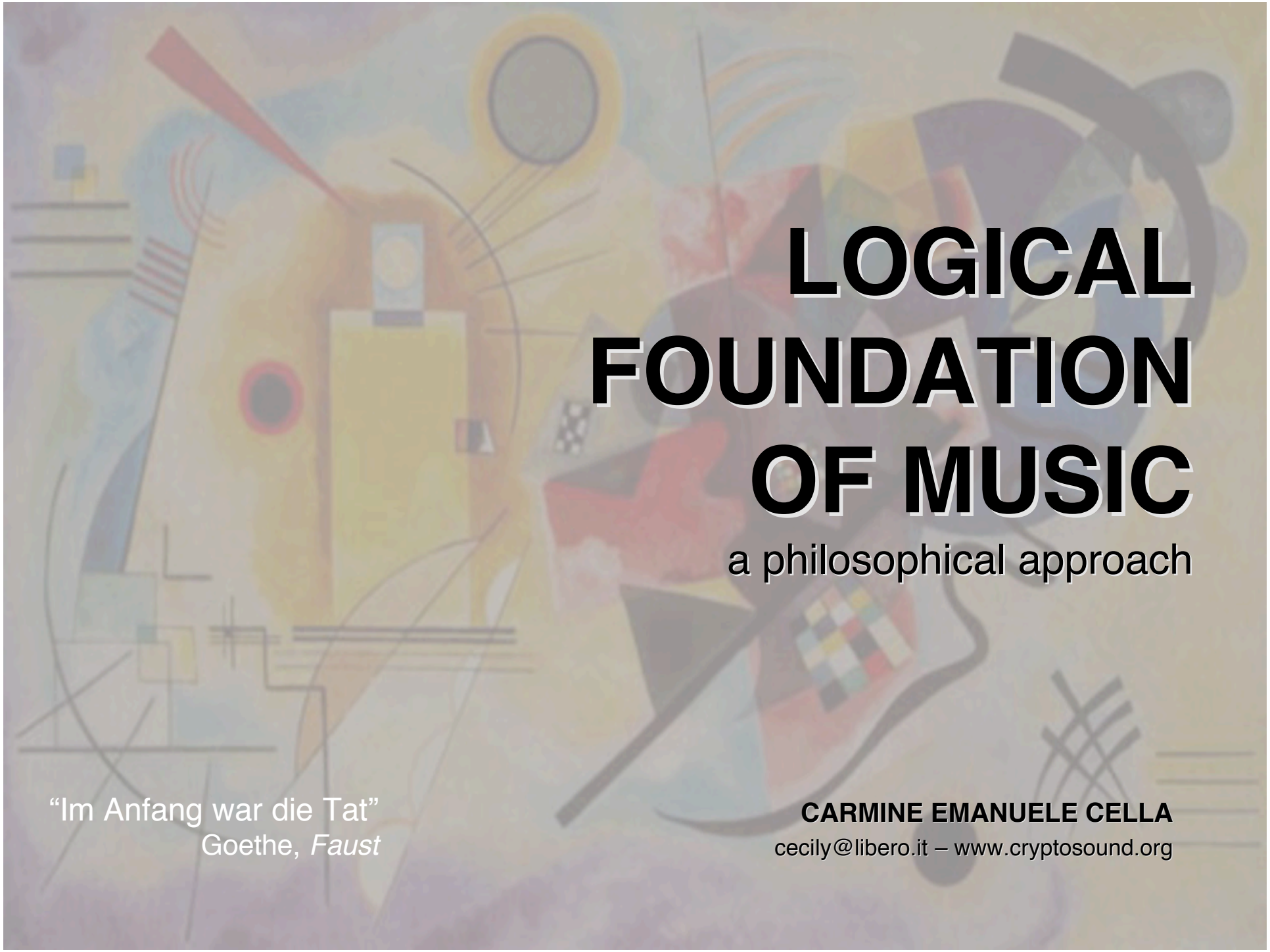

LOGICAL FOUNDATION OF MUSIC

a philosophical approach

"Im Anfang war die Tat"
Goethe, *Faust*

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NATURE OF MUSICAL KNOWLEDGE

- *Musical knowledge* can be thought as a complex system with a dual nature: **intuitive** and **formalized**
- Formalized nature is actually a *logical structure*, based on underlying **algebras** with well-structured **operators**
- Logical structures involved with music (*musical logics)* are *not only* truth-logics and don't belong to a single discipline
- Contributes to musical logics come from: philosophy, mathematics, artificial intelligence, musical theory, computer music, etc.

SUSANNE LANGER'S APPROACH (1)

- In 1929 the American review "*The Monist*" published a small article by Susanne K. Langer titled "**A set of postulates for the logical structure of music**"
- Every system has a finite number of possible configuration
- For relatively simple systems (for example the *chess game*) an exhaustive search for each configuration is possible, although difficult
- For complex systems however, this could be not possible (for example sciences, arts, etc.)

SUSANNE LANGER'S APPROACH (2)

- The only possible thing in such systems is to find **formal relations** among some basic elements
- Langer's hypothesis: **music is a system made of some basic elements linked by definite principles**
- A such set of principles constitutes the *abstract form* of the music or its *logical structure* and is itself a special algebra neither numerical nor Boolean but of equally mathematical form and amenable to at least **one** interpretation
- This logical structure is described by a set of **postulates**

BASIC POSTULATES (EXCERPTS)

- Let K be a set of elements, • and _ two binary operations, C a monadic relation (property) and < a diadic relation. Then hold:

1. If $a,b \in K \Rightarrow a \bullet b \in K$
2. $\forall a \in K \Rightarrow a \bullet a = a$
3. If $a \bullet b \in K \Rightarrow a \rightarrow b \in K$
4. $\forall a,b \in K, a \rightarrow b = b \rightarrow a \Rightarrow a = b$
5. $\forall a,b,c \in K, (a \bullet b) \bullet c = b \bullet (a \bullet c)$
6. $\forall a,b,c \in K, \exists \mathrm{d} \in \mathrm{K} / (a \rightarrow b) \bullet (c \rightarrow d) = (a \bullet c) \rightarrow (b \bullet d)$
7. $\exists r \in K / \forall a \in K, a \bullet r = a$

... etc....

MUSICAL INTERPRETATION (EXCERPTS)

- The interpretation of the described algebra leads to the creation of the **formal structure of music**:
  1. If *a, b* are **musical elements**, the **interval** a-*with*-b is a musical element
  2. If *a* is a musical element, the **unison** a-*with*-a is a musical element
  3. If *a, b* are musical elements, the **musical progression** a-*to*-b is a musical element
  4. If *a, b* are musical elements, and if a-*to*-b = b-*to*-a then *a* and *b* are the **same** musical element
  5. If *a, b, c* are musical elements then the interval (a-*with*-b)-*with*-c is the same interval of b-*with*-(a-*with*-c)
  6. If *a, b, c* are musical elements the exists at least a musical element *d* such as the interval of the progression (a-*to*-b)-*with*-(c-*to*-d) is equal to the progression of the interval (a-*with*-c)-*to*-(b-*with*-d) [**counterpoint principle**]

  … etc …

NOTES ON THE NEW ALGEBRA

- The postulates describe a new algebra that is **not** a Boolean algebra for the following reasons:
  1. _ it is non-commutative
  2. the *zero* of the algebra has an incomplete nature
  3. there isn't the *one* of the algebra
- All essential relations among musical elements can be demonstrated from the postulates, for example: the *repetitional character* of the order of tones within the octave, the *equivalence* of *consonance-values* of any interval and any repetition of itself, etc.

POSSIBLE EXTENSIONS

- Many other relations among musical elements can be derived from the postulate-set
- Even a complete development of it can give us only the *general* musical possibilities
- The structures employed in European music require further specifications as a *next-member* postulate for the series generated by <, determination of the consonant intervals other than unisons and repetitions, the introduction of T-function **#** and **b**, and so on.
- Alternative sets of restrictions upon original K can be used to derive different types of music (Hawaiian, Gaelic, etc.)

A SET-THEORETICAL APPROACH

- Langer's approach suffers from an overemphasis on harmony at the expense of contrapuntal texture
- It lacks of the **temporal dimension**: it's almost impossible to apply Langer's postulates to a *real world example*
- A more suitable approach involves set-theory
- Our concern will then be to take a few steps toward an adequate characterization of the musical system int set-theoretical terms: toward **abstract musical systems**

ABSTRACT MUSICAL SYSTEMS (1)

- A **temporal frame** is an oredered quadruple <T, t-, -t, ≤> satysfying the following axioms:

T1. T ≠ ∅	
T2. t-, -t ∈ T	
T3. t- ≠ -t	
T4. ≤ ∈ T X T	
T5. t- ≤ t	(t- ≤ - first in T)
T6. t ≤ -t	(-t ≤ - last in T)
T7. t ≤ t	(reflexivity in T of ≤)
T8. se t ≤ t' e t' ≤ t'' allora t ≤ t''	(transitivity in T of ≤)
T9. se t ≤ t' e t ' ≤ t allora t = t'	(anti-simmetry in T of ≤)
T10. t ≤ t' oppure t' ≤ t	(strong connexity in T of ≤)

ABSTRACT MUSICAL SYSTEMS (2)

- In the same way a **pitch frame** is an oredered quintuple

  <P, p-, -p, §, ≤> satysfying the same set of axiom P1-P10 obtained in perfect analogy with the set T1-T10 above, as well as the additional axiom:

  P11. § ∉ **P** (a null-pitch is not in P)

- A **musical frame** is a structure:

  <<T, t-, -t, ≤>, <P, p-, -p, §, ≤>, V> such as hold:

  (i). <T, t-, -t, ≤> is a temporal frame

  (ii). <P, p-, -p, §, ≤> is a pitch frame

  (iii). V is a non-empty set of “voices”

ABSTRACT MUSICAL SYSTEMS (3)

- A **musical frame** with **voice-indexed temporal partitions** is a structure:

  F = <<T, t-, -t, ≤>, <P, p-, -p, §, ≤>, V, S> such as hold:

  (i). <<T, t-, -t, ≤>, <P, p-, -p, §, ≤>, V> is a musical frame

  (ii). S is a "point-selector" over that frame in the sense of being a funcion from V to the power-set of T such as for each v ∈V:

  (ii.i). S_v is a *finite* subset of T

  (ii.ii) t- and -t are both in S_v

ABSTRACT MUSICAL SYSTEMS (4)

- Let F be a musical frame with voice-indexed temporal partitions. By a **melodic-rhythmic specification on F** we understand an ordered pair <On, FrAtt> of functions on V such as for each $v \in V$:

  (i). $On_v \subseteq T \times (P \cup \{\S\})$ ("on" function)

  (ii). $FrAtt_v \subseteq T \times (P \cup \{\S\})$ ("freshly attacked" func.)

  NB: The pair must satisfy also a special set of axioms MR1-5

ABSTRACT MUSICAL SYSTEMS (5)

- By an **abstract musical system** we now understand a structure

  M = <F, <On, FrAtt>> such as:

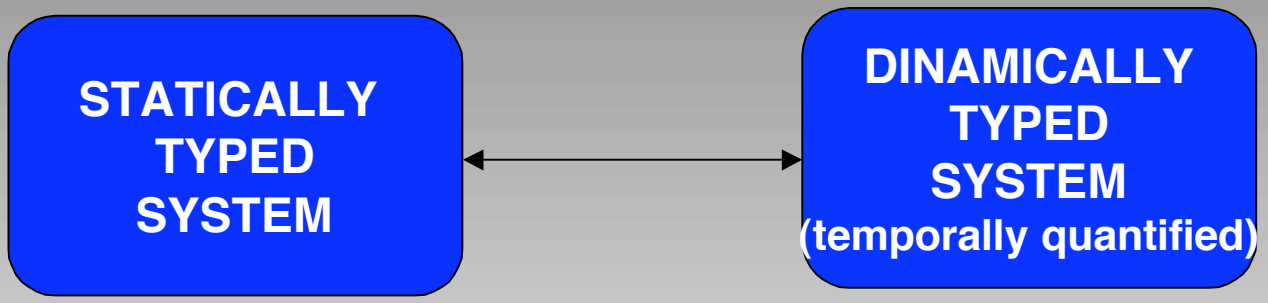
  (i). F is a musical frame with voice-indexed temporal partitions

  (ii). <On, FrAtt> is a melodic-rhythmic spec. on F
- With the same formalism we can define also: the **musical course of events in v in M (mce)**, the **texture of M (Texture)**, and the **total chord progression in M (**Chord)
- Finally: *counterpoint* is the study of Texture structure while *harmony* is the study of Chord structure

DIFFERENT POINTS OF VIEW

Langer' postulates

Set-theoretical a. m. s.



A PHILOSOPHICAL PERSPECTIVE

- In 1910 E. Cassirer (1874-1945) published an essay titled "*Substanzbegriff und Funktionsbegriff*" (Substance and function)
- Through a solid acquaintance of history of science, Cassirer conducts an inquiry into mathematical, geometric, and physical knowledge
- Cassirer shows how these different forms of knowledge don't look for the common (*substance*) but for the general laws, the relations (*functions)*
- Scientific knowledge leads us to move from the concept of substance to the concept of function

A-PRIORI KNOWLEDGE

- Mathematical functions *are not* abstractions from substances but are created by thought
- In the same way, *scientific theories* and *functional relations* among knowledge objects are created by thought
- The knowledge is **a-priori**: the human *act* of knowing is the milestone of knowledge and not the substance *per sè*
- In this sense the human being is **animal symbolicum**

SUPREMACY OF ACTION

- Cassirer's ideas on substance/function duality have roots in the philosophy of Paul Natorp (1854-1924), a former Cassirer's teacher
- Following Natorp, **reality is not made by the objects discovered by knowledge but is the same *discovering process***
- We move from the *structure* to the *process (action)*
- Natorp quotes Goethe: "Im Anfang war die Tat" (*At the beginning there was the Action)*

THE SIMPLE SYSTEM (INFORMALLY)

- Music can be thought as a *simple system* organized into two distinct categories: **state** and **transition**
- A *state* is an **ideal** configuration in which the parameters of music are in *rest*
- A *transition,* on the contrary, is a **possible** configuration in which the parameters are in *tension*, continuously evolving
- Following Cassirer, the former can be thought as *substance*, the latter as *function*

THE GENERATION FUNCTION (INFORMALLY)

- Let be S_1 and S_2 two different states. Then we can define a function Φ: S_1 _ S_2 called *generator,* such as:

  (i). Φ creates a *transformation* of S_1 into S_2 throught a finite number of steps called *orbits* (temporal evolution)

  (ii). Φ holds for each parameter of the musical system, such as melody, harmony and rhythm
- **It is very important to think music as a dinamically-typed system, by defining proper generators for each needed parameter**

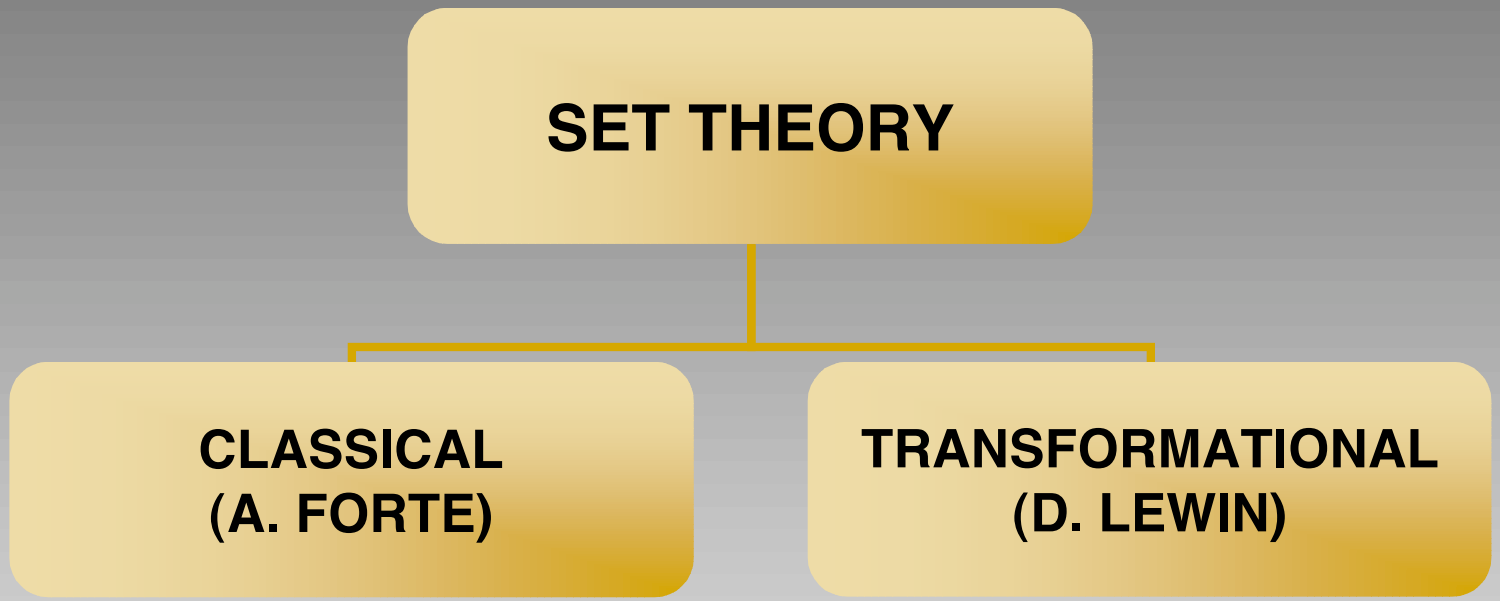
MELODIC REGIONS

- Let be S the set of the twelve distinct pitch-classes. Then P_0, P_1, …, P_n will be called a **special ordering** of S.
- Φ is a *permutation* from P_n to P_{n+1}
- Each P_n is a *state* while the *orbits* created by Φ are *transitions*
- The whole set of transitions will be called **melodic region**

HARMONIC REGIONS

- Let O be a set of distinct pitch-classes, called *orbit*.
- If some elements of O occurs *simultaneously* the O will be called **harmonic field**
- Every orbit can have a finite number of harmonic fields; the set of fields of a single orbit is called **harmonic orbit**
- The set of the harmonic horbits will be called **harmonic region**
- A single pitch orbit is an harmonic *transition*, while a field is a *state*
- Harmony and melody will *never* be in the same configuration

LEWIN'S PERSPECTIVE

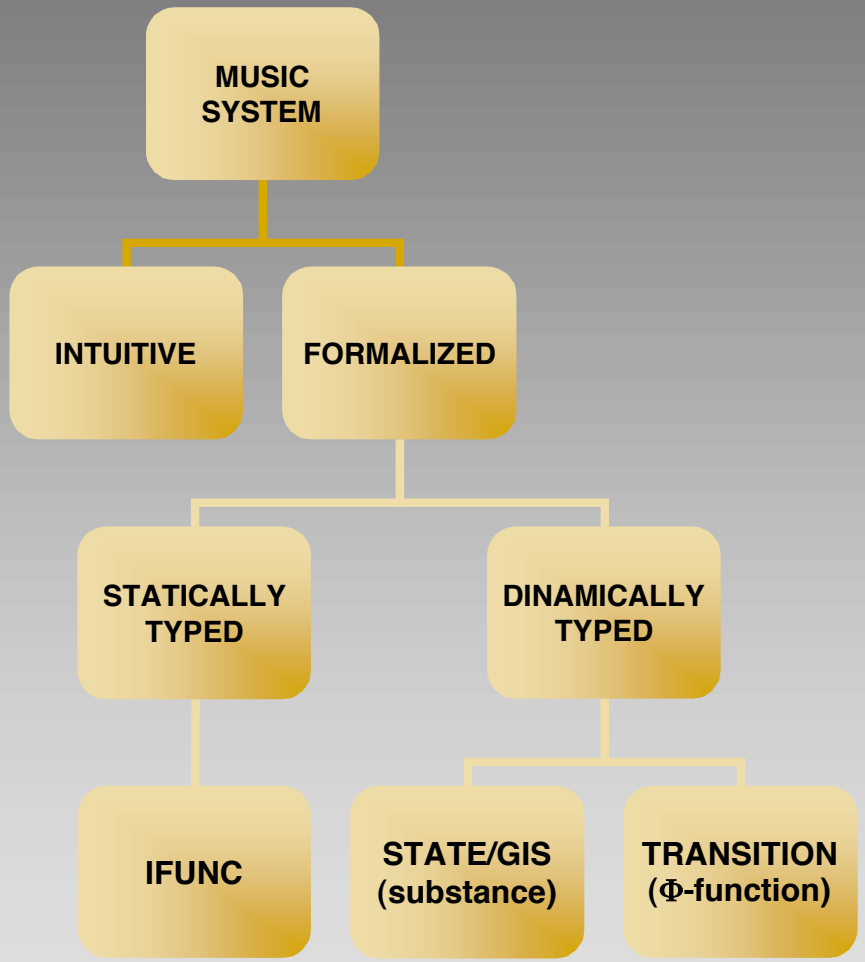


- Music can be *represented* through a formal structure called **GIS** (Generalized Interval System) and through a *transformation* function called **IFUNC** (Interval function)

CLOSING THE CIRCLE

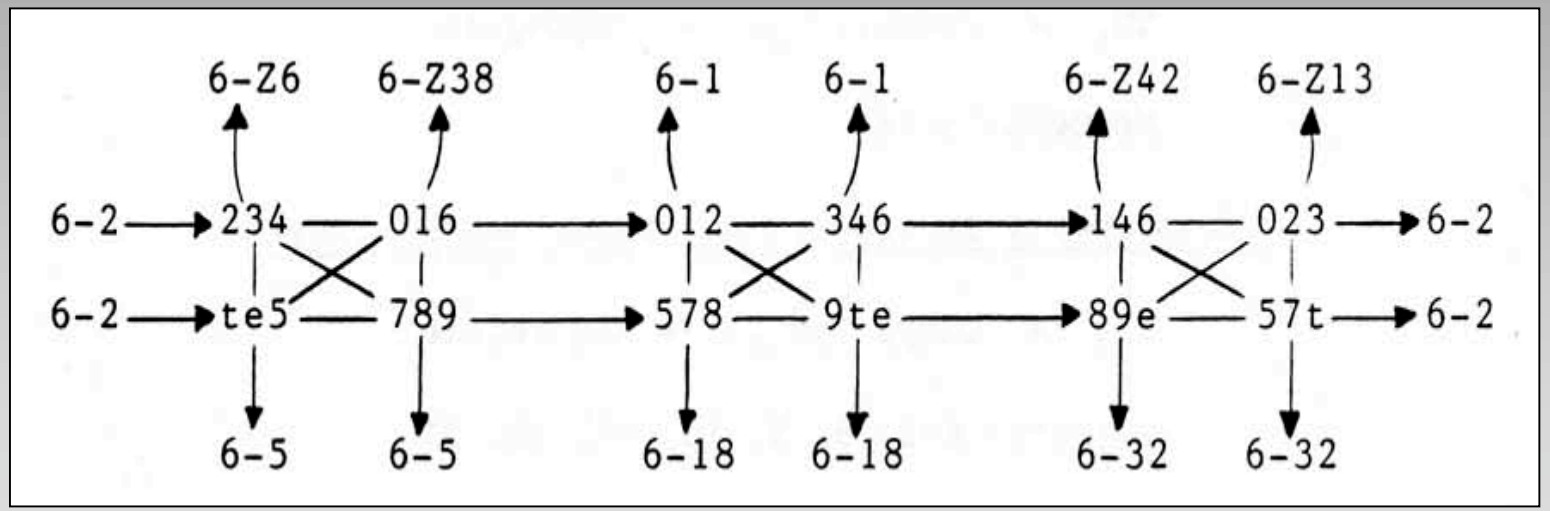
- A **GIS** can be thought as a *state?*
- The **IFUNC** can be thought as a *transition?*
- Φ (*generator*) must hold for all the parameters in the system and must *happen* in a temporal frame
- Does IFUNC satisfy these requirements?

A VISUAL SUMMARY



MUSICAL EXAMPLES

- **Vectorial synthesis** from two sets of partials in additive synthesis (SineWarp 1.0)
- **Trichordal generators** of hexachords as explained by Steve Rouse in 1985:



(excerpts from “Paracelso y la rosa”, 2005)

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